Algebraic foliations and dicritical divisors

Conferencia dentro del Semestre IMAC en Sistemas Dinámicos

21 de junio de 2011

IMAC, Universitat Jaume I de Castelló, Spain.

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A classical problem, proposed by H. Poincaré, consists of determining whether a differential equation with polynomial coeficients, of first order and degree 1, is algebraically integrable or, equivalently, whether an algebraic foliation of the projective plane has a rational first integral. We shall show that, assuming certain conditions related with the dicritical divisors that appear in the resolution of singularities, there exists an algorithm that provides a solution to this problem whose input is the resolution of the (dicritical) singularities of the foliation. We shall see also that these assumed conditions are not necessary if there is only one dicritical divisor. In addition, we shall show that, when the number of dicritical divisors is two, it is possible to determine (without the additional conditions) whether a foliation has a rational first integral of fixed genus.